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The problem of finding a finite dimensional suboptimal controller is also studied in this project. For the SISO case, it is shown that by approximating the infinite dimensional parts of the optimal controller (with certain interpolation constraints), a rational suboptimal controller can be obtained.

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# Controller Design for
# Unstable Aeroelastic Systems

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# OVERVIEW

In this report we describe the research performed at The Ohio State University on the project *Controller Design for Unstable Aeroelastic Systems*, sponsored by the Air Force Office of Scientific Research (AFOSR), between June 15, 1993 and January 14, 1995, under contract F49620-93-1-0288.

The main objective of this research was the robust controller design for active flutter suppression, in the presence of time delays and neglected dynamics in the feedback loop. For this purpose we have developed an algorithm (and implemented on the computer) which derives a set of finitely many linear equations and computes the $H^\infty$ optimal and suboptimal controllers for unstable systems with time delays. With this algorithm we can now compute the effect of time delays on the best achievable robustness and performance level in unstable aeroelastic systems. Moreover, all suboptimal $H^\infty$ controllers are computed for a given system with time delay and dynamic uncertainty profile. Our algorithm determines the "rational parts" of suboptimal $H^\infty$ controllers. Such controllers contain an infinite dimensional fixed part, which is the given time delay. Therefore, the controller is "infinite dimensional."

The problem of finding a finite dimensional suboptimal controller is also studied in this project. For the SISO case, it is shown that by approximating the infinite dimensional parts of the optimal controller (with certain interpolation constraints), a rational suboptimal controller can be obtained.

In this project we have also performed robustness analyses of controllers designed for active flutter suppression. Several different robust control algorithms are compared in terms of the robustness of stability with respect to perturbations in the air speed and unmodeled small time delays. Our numerical studies on a thin airfoil example showed that the best results are obtained when weighted gap optimization using $H^\infty$ optimal control, and $\mu$-synthesis methods are employed. Also, studied in the project is the computational complexity of certain robust control algorithms, such as $\mu$-analysis and synthesis, simultaneous stabilization, and solution to bilinear matrix inequalities which appear in $H^\infty$, $H^2$ and mixed types of robust control problems. We have shown that many of these problems are NP-hard, and hence it is not feasible to develop a polynomial time algorithm to find exact solutions to these problems.

The principal investigator of this project was Hitay Özbay. One doctoral student, Onur Toker (expected graduation is Summer 1995), was supported by this grant, and his contributions were as much as the PI's.

Four journal papers, based on this research, are accepted for publication and three others are under review. Three of the accepted papers are co-authored by H. Özbay and O. Toker, and one of them is single authored by O. Toker. Preliminary versions of these papers already appeared or accepted for publication in conference proceedings. A list of publications resulting from this research are given in the last section of this report.

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# Contents

# 1   Introduction

The main goal of this project was to investigate robust control algorithms for unstable aeroelastic systems, against unmodeled dynamics (mainly due to poor modeling in aeroelastic loads) and in the presence of time delays in the feedback loop.

A robust control algorithm has been developed for general MIMO unstable systems with time delays. This methods follows the gap metric approach for uncertain systems. Also, the general theory for $H^\infty$ control of infinite dimensional SISO plants have been simplified. The controller expression obtained in this project is so simple that, computation of the optimal and all suboptimal controllers can be done by using *finite dimensional linear algebra*, see [23], [10]. Implementation issues for such controllers have been discussed, and an approximation procedure has been proposed in [30], [29].

Several robust control algorithms have been tested for active flutter suppression of a thin airfoil, see [26]. In this study it has been shown that even small time delays (not considered in controller design) may cause instability, or poor performance. The MIMO gap metric design algorithm developed in this project has also been applied to this system. The results show closed loop stability with guaranteed robustness measure, with respect to gap metric. But the controller itself contains time delays and its finite dimensional parts are very large order. Nevertheless the Bode plots of the controller indicate that it is easy to find a low order finite dimensional controller which will do the job, see Section 4 below.

Computational issues of several robust control algorithms are also investigated; and it is shown that certain analysis and synthesis problems computationally are not feasible, see [27], [25], [24], and Section 5 below for precise statements of the results.

The rest of this report is organized as follows. In Section 2 robust control problems for aeroelastic systems are reviewed. In Section 3 control algorithm for MIMO time delay systems is presented. Applications of different robust control techniques to active flutter suppression are given in Section 4. In Section 5 certain results concerning the computational issues in robust control are summarized. Concluding remarks are made in Section 6. A list of publications based on this project can be found in the last section.

# 2    Robust Control of Aeroelastic Systems

The most common approach for modeling large scale structures is to use linear finite element (FE) methods. In aeroelastic systems, where there is coupling between the structure and the air flow, dynamic aeroelasticity equation can be written as (see [1], [4], [12], [13], etc.),

$$M\ddot{x} + C\dot{x} + Kx = F_{\text{aero}}(\dot{x}, x) + F_{\text{ext}} \tag{1}$$

where $x$ is the FE displacement vector, $M, C, K$ are the mass, structural damping and stiffness matrices respectively; $F_{\text{aero}}$ and $F_{\text{ext}}$ represent the aerodynamic forces and the external (e.g. control, disturbance, noise, etc.) forces respectiveley.

In flutter analysis one looks for singularities of the equation (1), assuming $F_{\text{ext}} = 0$. In the flutter suppression problem the purpose is to choose (if possible) $F_{\text{ext}}$ as a feedback, i.e. $F_{\text{ext}}(x)$ is an operator acting on $x$, such that the closed loop system is stable. The key difficulty in these problems is the computation of $F_{\text{aero}}$, which, in theory, should be an infinite dimensional nonlinear operator. In practice, $F_{\text{aero}}$ is expressed as an output of a linear system whose input is $x$: in the frequency domain we have

$$F_{\text{aero}}(j\omega) = H(j\omega)X(j\omega) \tag{2}$$

where $H(j\omega)$ is determined from the geometry of the structure, and the aerodynamic conditions (in particular the air speed $V$), see e.g. [5] [6] [19] [12] [13], etc. In order to determine the flutter speed we need to analyze the location of the eigenvalues of the matrix

$$A(s) := Ms^2 + Cs + K - H(s)$$

as $V$ changes. Flutter occurs at the air speed $V = V_f$, which makes $A(j\omega)$ singular. In other words $V_f$ is the critical air speed at which the aeroelastic system becomes dynamically unstable.

Since $H(s)$ is impossible to determine precisely, in practice an approximate, say $H_n(s)$, is used in the flutter analysis, and suppression problems. Hence, neglecting approximation error and parametric uncertainties, which may occur in modeling $M$,
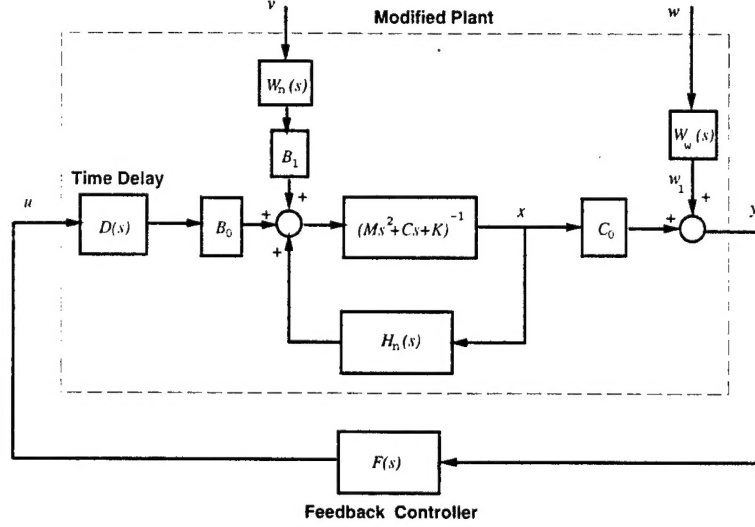
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Figure 1: Aeroelastic system model with time delay

$C$, $K$, and assuming zero initial conditions and taking the Laplace transforms of both sides of (1) one gets

$$X(s) = \left(Ms^2 + Cs + K - H_n(s)\right)^{-1} F_{\text{ext}}(s) \tag{3}$$

as the nominal model to be used in control design, and stability analysis. For feedback controller design purposes we can think that $F_{\text{ext}}(t) = B_0 u(t)$, where the vector $u(t)$ represent the command inputs, which are the outputs of a controller, whose inputs are the measured outputs. The mesured outputs are represented in one vector $y(t)$, given by

$$y(t) = C_0 x(t) + w_1(t),$$

where $w_1(t)$ is the measurement noise. The noise is assumed to be the output of a filter $W_w(s)$ whose input is a finite energy signal $w(t)$. Here $B_0$, $B_1$, $C_0$, are appropriate size matrices. A block diagram of this aeroelastic system model is shown in Figure 1, where $D(s)$ represents possible time delays in the feedback loop, and $B_1 W_n(s)$ can be seen as the actuator disturbance or an artificial noise which accounts for unmodeled dynamics.

In the literature, several different control schemes for active flutter suppression have been reported, see e.g. [9] and references therein. But none of these controllers
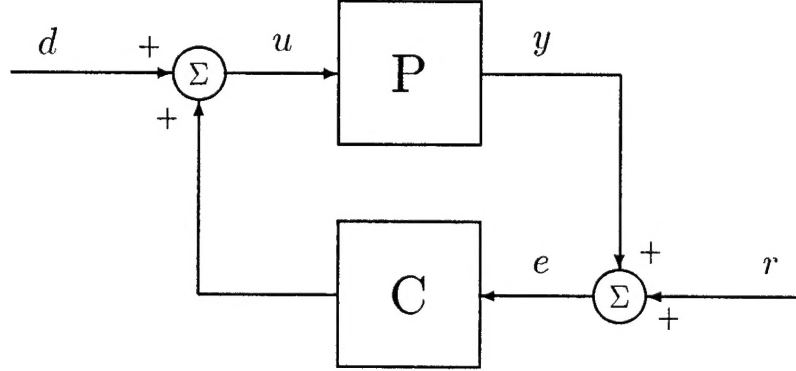
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Figure 2: Standard feedback configuration

take into account the unmodeled dynamics or possible time delays in the feedback loop. In the following section feedback controller design for this type of input/output models of the aeroelastic system will be discussed.

# 3    A Robust Control Algorithm for MIMO Time Delay Systems

Now, the aeroelastic plant model developed in the previous section can be written as

$$P(s) = e^{-Ts}P_o(s),$$

where $e^{-Ts} = D(s)$ is the delay, and

$$P_o(s) = C_o(Ms^2 + Cs + K - H_n(s))^{-1}B_o$$

is a strictly proper rational $m \times n$ transfer matrix. The feedback control system is shown in Figure 3, (here $C(s)$ is the controller, and $P(s)$ is the plant, $r$ is the reference input, $d$ is the disturbance). The closed loop system $[P, C]$ is said to be stable if the entries of all transfer function matrices $(I - CP)^{-1}$, $(I - CP)^{-1}C$, $P(I - CP)^{-1}$, and

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$P(I - CP)^{-1}C$ belong to $H^\infty$. When the closed loop system is stable we can define

$$b_{P,C} = \left\| \begin{bmatrix} I \\ P \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} I & -C \end{bmatrix} \right\|_\infty^{-1}$$

as the stability robustness level of the system $[P, C]$. Larger the $b_{P,C}$ more robustness we have against unmodeled normalized coprime factor perturbations of $P$, see [8]. With this definition, closed loop systems $[P_\delta, C]$ are stable for all $P_\delta$, which belongs to a gap ball of radius $\delta$ around $P$, if and only if $\delta < b_{P,C}$. Hence, for a given plant $P$, the optimal robustness radius can be defined as

$$b_{opt}(P) := \sup_{C \text{ stabilizes } P} b_{P,C}.$$

In [18], it is shown that, $b_{opt}(P)$ can be computed using state space techniques. The above problem is called the gap metric optimization problem, and has another interpretation in terms of the (unweighted) coprime factor perturbations of the original plant. Namely, if $P = NM^{-1}$ is a normalized coprime factorization, the above optimization problem corresponds to finding the maximum $\epsilon$ such that there exists a controller $C$ which stabilize all $P_\Delta = (N + \Delta_N)(M + \Delta_M)^{-1}$ with $\| \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix} \|_\infty < \epsilon$. This maximum $\epsilon$ value is equal to $b_{opt}(P)$.

The suboptimal robustness problem is to find a parameterization of the set

$$\mathcal{C}_\gamma = \left\{ C \ : \ [P, C] \text{ stable }, \ b_{P,C} \geq \gamma \right\}. \tag{4}$$

This problem is equivalent to parameterizing all controllers $C$ which robustly stabilizes $P$ against all coprime factor perturbations of the form

$$\| \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix} \|_\infty < \gamma =: \frac{1}{\sqrt{\rho^2 + 1}}$$

for a given desired robustness level $\gamma$. The main result of this section is a parameterization of $\mathcal{C}_\gamma$ obtained in terms of the state space realizations of $P_o$. The algorithm summarized below has been implemented using MATLAB commands.

## Outline of the procedure:

Let $(A_o, B, C)$ be a minimal realization of $P_o(s)$, then find the stabilizing solution $R_F$ of the algebraic Riccati equation

$$A_o R_F + R_F A_o^* - R_F C^* C R_F + BB^* = 0 \tag{5}$$

and let $A = A_o + HC$ where $H = -R_F C^*$. Then

$$F(s) := \left[ -\tilde{N}, \tilde{M} \right] = \left[ -C(sI - A)^{-1} B e^{-Ts}, I_m + C(sI - A)^{-1} H \right].$$

Similarly, find the stabilizing solution $R_G$ of the algebraic Riccati equation

$$A_o^* R_G + R_G A_o - R_G BB^* R_G + C^* C = 0 \tag{6}$$

and let $A_G = A_o + BH_G$ where $H_G = -B^* R_G$. Then

$$G(s) := \begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} I_n + H_G(sI - A_G)^{-1} B \\ C(sI - A_G)^{-1} B e^{-Ts} \end{bmatrix}.$$

Since $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$ is a coprime factorization (in fact normalized coprime factorization), there exist $U, V, \hat{U}, \hat{V} \in H^\infty$ such that the generalized Bezout equation

$$\begin{bmatrix} \hat{V} & -\hat{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U \\ N & V \end{bmatrix} = \begin{bmatrix} M & U \\ N & V \end{bmatrix} \begin{bmatrix} \hat{V} & -\hat{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ 0 & I_m \end{bmatrix} \tag{7}$$

holds. To parameterize the set of all suboptimal controllers, first choose a positive real number $a$. Then, let $x_o^1, y_o^1 \in \mathcal{H}_2$ be the solution of

$$\mathbf{\Gamma}_s x_o = -\rho y_o^* \quad \text{and} \quad \mathbf{\Gamma}_s^* y_o^* = -\rho x_o + \frac{I_m}{\rho} \frac{1}{s+a}, \tag{8}$$

and $x_o^2, y_o^2 \in \mathcal{H}_2$ be the solution of

$$\mathbf{\Gamma}_s^* x_o^* = -\rho y_o \quad \text{and} \quad \mathbf{\Gamma}_s y_o = -\rho x_o^* - \frac{I_n}{\rho} \frac{1}{s-a}, \tag{9}$$

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where the Hankel operator $\mathbf{\Gamma}_s$ is defined as

$$\mathbf{\Gamma}_s = \mathbf{\Pi}_{H_2^\perp} G^* \begin{bmatrix} U \\ V \end{bmatrix} |_{H_2} \quad.$$

In [22], we discussed how to obtain numerical solutions of (8,9) using state space techniques.

Given numerical solutions of (8,9), define

$$G^1_{AAK} = [2ax^1_o(a)]^{-1/2} >> 0 \qquad\qquad G^2_{AAK} = [2a\left(x^2_o(a)\right)^T]^{-1/2} >> 0$$

$$P^1_{AAK}(s) = \rho(s+a)x^1_o(s)G^1_{AAK} \qquad\qquad P^2_{AAK}(s) = \rho(s+a)\left(x^2_o(s)\right)^T G^2_{AAK}$$

$$Q^1_{AAK}(s) = \rho(s-a)\left(y^1_o(s)\right)^T G^1_{AAK} \qquad\qquad Q^2_{AAK}(s) = \rho(s-a)y^2_o(s)G^2_{AAK}$$

where $>>$ stands for positive definite square root.

Then, the set of all suboptimal controllers can be parameterized as :

$$\mathcal{C}_\gamma = \{[N_{c,1}(s) + N_{c,2}(s)\mathcal{E}(s)][D_{c,1}(s) + D_{c,2}(s)\mathcal{E}(s)]^{-1} \quad : \quad \mathcal{E} \in H^\infty \quad \|\mathcal{E}\|_\infty \leq 1\},$$

where

$$\begin{aligned}
N_{c,1}(s) &= [\rho M(s)Q^1_{AAK}(-s) - \tilde{N}^*(s)P^1_{AAK}(s)] \\
N_{c,2}(s) &= [\rho M(s)P^2_{AAK}(-s) - \tilde{N}^*(s)Q^2_{AAK}(s)] \\
D_{c,1}(s) &= [\rho N(s)Q^1_{AAK}(-s) + \tilde{M}^*(s)P^1_{AAK}(s)] \\
D_{c,2}(s) &= [\rho N(s)P^2_{AAK}(-s) + \tilde{M}^*(s)Q^2_{AAK}(s)].
\end{aligned}$$

Note that all suboptimal controllers are expressed as linear fractional transformation of $\mathcal{E}$, with coefficients determined by $N, M, \tilde{N}, \tilde{N}$ and solutions of (8,9).

In the next section the Bode plots of a controller obtained from the above procedure is presented for an active flutter suppression problem. For the same problem, results of different robust control algorithms are also given in the next section.
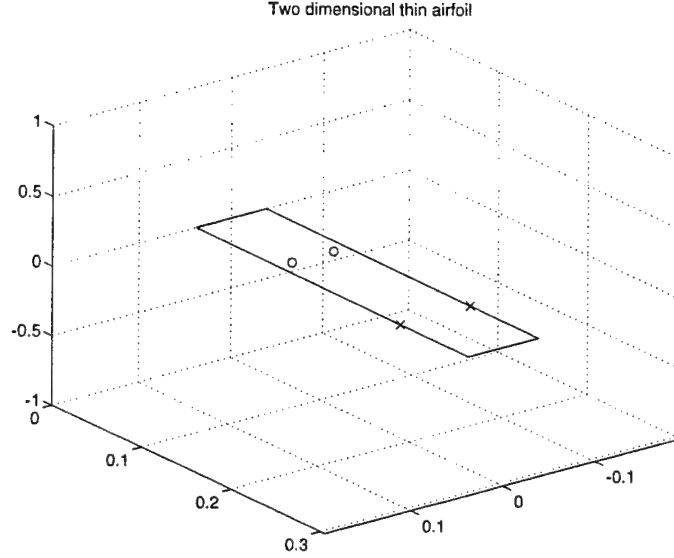
Figure 3: Two dimensional thin airfoil, 'o' observation points, 'x' actuator locations

# 4    Applications to Active Flutter Suppression

The purpose of this section is to give a comparison of different robust controller design techniques for a linear model of a two dimensional thin airfoil described in [7].

## 4.1    Mathematical Model of the Plant

Consider the two dimensional thin airfoil, of length $\ell = 0.305m$ and with $c = 0.076m$, shown in Figure 3. In this system two point force actuators are located at $x = 3\ell/4$, $y = \pm c/2$, and vertical displacements $w(x, y, t)$ at $x = 0.3\ell$, $y = \pm 0.3c$ are assumed to be available for feedback control.

The mathematical model of this system is taken from [7], and it is as given below.

$$[M - \frac{\pi \rho c^2}{4} M_a]\frac{d^2 q(t)}{dt^2} - \pi \rho c V C_a \frac{dq(t)}{dt} + [K - \pi \rho V^2 K_a]q(t)$$

$$-\pi \rho c V D_1 \eta_1(t) - \pi \rho c V D_2 \eta_2(t) - \Gamma^T F(t) = 0, \quad (10)$$

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$$\frac{d\eta_1(t)}{dt} = -0.041\frac{2V}{c}I_5\eta_1(t) - A_4\frac{d^2q(t)}{dt^2} + \frac{V}{c}A_5\frac{dq(t)}{dt}, \tag{11}$$

$$\frac{d\eta_2(t)}{dt} = -0.032\frac{2V}{c}I_5\eta_2(t) - A_4\frac{d^2q(t)}{dt^2} + \frac{V}{c}A_5\frac{dq(t)}{dt}, \tag{12}$$

where the entries of $2 \times 1$ vector

$$F(t) = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix},$$

represent the control forces (to be generated by the feedback controller), the entries of $5 \times 1$ vector $q(t)$ are the generalized coordinates, and the entries of $5 \times 1$ vectors $\eta_1(t)$, $\eta_2(t)$ are the generalized aerodynamic coordinate vectors. The nominal value of the air speed (in the direction of $y$-axis) is taken to be $V = 30m/sec$, and the other constants are

$$\begin{aligned}
\rho &= 1.225 \text{ kg/m}^3 \\
c &= 0.076 \text{ m} \\
\ell &= 0.305 \text{ m}
\end{aligned}$$

$$\Gamma^T = \begin{bmatrix} 1.315 & 1.315 \\ 0.270 & 0.270 \\ 0.631 & -0.631 \\ -0.081 & -0.815 \\ 0.800 & 0.800 \end{bmatrix},$$

$$K_a = \begin{bmatrix} 0.000 & 0.000 & 0.289 & -0.015 & -0.444 \\ 0.000 & 0.000 & 0.015 & 0.294 & -0.541 \\ 0.000 & 0.000 & 0.069 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.072 & 0.000 \\ 0.000 & 0.000 & 0.037 & 0.042 & 0.681 \end{bmatrix},$$

$$C_a = \begin{bmatrix} -0.305 & 0.000 & 0.144 & -0.007 & -0.370 \\ 0.000 & -0.305 & 0.007 & 0.147 & -0.451 \\ -0.072 & -0.037 & 0.000 & 0.000 & 0.186 \\ 0.004 & -0.074 & 0.000 & 0.000 & 0.209 \\ -0.037 & -0.045 & -0.009 & -0.010 & -0.114 \end{bmatrix},$$

$$D_1 = -0.165I_5, \quad D_2 = -0.355I_5,$$

$$M = \begin{bmatrix} 0.0283 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0283 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0021 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0021 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0034 \end{bmatrix},$$

$$M_a = \begin{bmatrix} -0.3050 & 0.0000 & 0.0000 & 0.0000 & 0.0190 \\ 0.0000 & -0.3050 & 0.0000 & 0.0000 & 0.0230 \\ 0.0000 & 0.0000 & -0.0090 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.0090 & 0.0000 \\ 0.0190 & 0.0230 & 0.0000 & 0.0000 & -0.1140 \end{bmatrix},$$

$$K = 10^4 \begin{bmatrix} 0.0121 & 0.0000 & -0.0046 & -0.0056 & 0.0328 \\ 0.0000 & 0.4749 & 0.0332 & -0.0573 & -0.2502 \\ -0.0046 & 0.0332 & 0.0326 & -0.0452 & -0.0691 \\ -0.0056 & -0.0573 & -0.0452 & 0.3012 & 0.0268 \\ 0.0328 & -0.2502 & -0.0691 & 0.0268 & 3.3105 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \frac{1}{4}\phi_3 \\ \frac{1}{4}\phi_4 \\ \frac{1}{6}\phi_5 \end{bmatrix} \begin{bmatrix} \phi_1 & \phi_2 & -\frac{1}{4}\phi_3 & -\frac{1}{4}\phi_4 & \frac{1}{6}\phi_5 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \frac{1}{4}\phi_3 \\ \frac{1}{4}\phi_4 \\ \frac{1}{6}\phi_5 \end{bmatrix} \begin{bmatrix} 0 & 0 & \phi_3 & \phi_4 & -2\phi_5 \end{bmatrix},$$

$$\phi_1 = \cosh(\frac{n_1}{\ell}x) - \cos(\frac{n_1}{\ell}) - 0.7340955 \left[ \sinh(\frac{n_1}{\ell}x) - \sin(\frac{n_1}{\ell}x) \right],$$

$$\phi_2 = \cosh(\frac{n_2}{\ell}x) - \cos(\frac{n_2}{\ell}) - 1.01846644 \left[ \sinh(\frac{n_2}{\ell}x) - \sin(\frac{n_2}{\ell}x) \right],$$

$$\phi_3 = \sin(g_1 x) - \frac{g_1}{f_1} \sinh(f_1 x) + \left( \frac{g_1^2 \sin(g_1 \ell) + g_2 f_1 \sinh(f_1 \ell)}{g_1^2 \cos(g_1 \ell) + f_1^2 \cosh(f_2 \ell)} \right) \left( \cosh(f_1 x) - \cos(g_1 x) \right),$$

$$\phi_4 = \sin(g_2 x) - \frac{g_2}{f_2} \sinh(f_2 x) + \left( \frac{g_2^2 \sin(g_2 \ell) + g_2 f_2 \sinh(f_2 \ell)}{g_2^2 \cos(g_2 \ell) + f_2^2 \cosh(f_2 \ell)} \right) \left( \cosh(f_2 x) - \cosh(g_2 x) \right),$$

$$\phi_5 = 1 - \cos(\frac{2\pi}{\ell} x),$$

where

$$
\begin{aligned}
n_1 &= 1.8751041 \\
n_2 &= 4.69409113 \\
f_1 &= 9.193529 \\
f_2 &= 17.022282 \\
g_1 &= 6.373793 \\
g_2 &= 15.679999
\end{aligned}
$$

The measured outputs, $y_1$ and $y_2$, are vertical displacements at the observation points indicated by 'o' in Figure 3, i.e.

$$y_1(t) = w(0.3\ell, -0.3c, t), \qquad y_2(t) = w(0.3\ell, 0.3c, t)$$

where

$$w(x, y, t) = \sum_{i=1}^{5} \gamma_i(x, y) q_i(t),$$

$$\gamma_i(x, y) = \phi_i(x) \epsilon_i(y)$$

$$
\begin{aligned}
\epsilon_1(y) &= & \epsilon_2(y) &= 1 \\
\epsilon_3(y) &= & \epsilon_4(y) &= y/c \\
\epsilon_5(y) &= 4y^2/c^2 - 1/3
\end{aligned}
$$

In general, for point force actuators located at $(x_1, y_1), \ldots, (x_p, y_p)$, the matrix $\Gamma$ is given by

$$\Gamma = \begin{bmatrix} \gamma_1(x_1, y_1) & \cdots & \gamma_5(x_1, y_1) \\ \vdots & \cdots & \vdots \\ \gamma_1(x_p, y_p) & \cdots & \gamma_5(x_p, y_p) \end{bmatrix}.$$

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## 4.2  Flutter Comparisons

Now consider the plant described above with transfer matrix $P_o(s)$, which depends on $V$ (in this section we assume that there is no time delay is the system). The open loop system and the closed loop system stability is analyzed, and the results are shown in the following figures. We see that for the open loop system the flutter speed is $V_f = 17.2 m/sec$. We designed controller for the nominal air speed $V = 30 m/sec$, which means that the open loop system is unstable at this operating point. The figures 5–9 illustrate stability robustness with respect to perturbations in $V$, for controllers designed using LQG, unweighted and weighted gap, $H^\infty$ mixed sensitivity, and $\mu$ synthesis techniques. The results can be summarized as follows:

| controller | admissible V (m/sec) |
|---|---|
| 0 (open loop) | $0.00 < V < 17.20$ |
| LQG | $5.37 < V < 30.49$ |
| gap (unweighted) | $29.51 < V < 30.22$ |
| gap (weighted) | $11.89 < V < 40.01$ |
| $H^\infty$ (mixed sensitivity) | $10.35 < V < 40.15$ |
| $\mu$ synthesis | $10.27 < V < 40.24$ |

The optimal controller guarantees robust stability for all plants of the form $P(s) = P_o(s) + \Delta_a(s)$, where the uncertainty is bounded by $r_a(j\omega) > \|\Delta_a(j\omega)\|$, i.e. $r_a(j\omega)$ can be seen as a measure of the stability margin. For the flutter suppression problem considered here. The plot of the largest allowable additive uncertainty magnitude, $r_a(j\omega)$, versus $\omega$ is shown in Figure 10, for two different time delays $1msec$ and $10msec$, for three different nominal $V$, $30m/sec$, $50m/sec$ and $200m/sec$. As the time delay and $V$ increases the robustness level decreases almost uniformly in the frequency spectrum. The additive uncertainty is allowed to be large in the frequency range $1rd/sec$ to $100rd/sec$.

## 4.3  Effects of Time Delays

Consider the controllers $C_{H\infty}$ and $C_{LQG}$ obtained from the $H^\infty$ and LQG designs when a time delay of 1msec is neglected. Time domain responses (the initial, $t = 0$, and the final, $t = 4.0sec$, shapes of the structure) to a unit gust disturbance are shown in Figures 11 and 12, for the closed loop system with no delay and 1msec delay. We see that the $H^\infty$ (LQG) controller is (is not) robust to unmodeled time delay of 1msec.
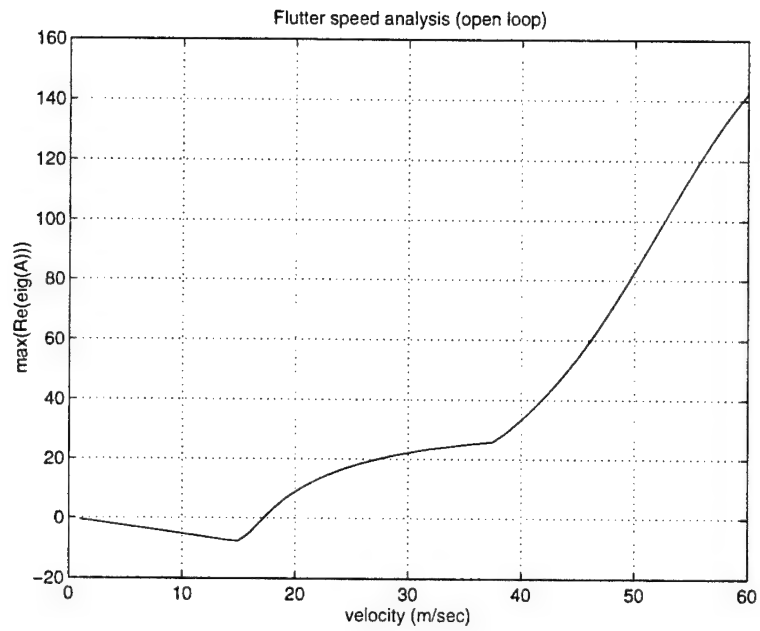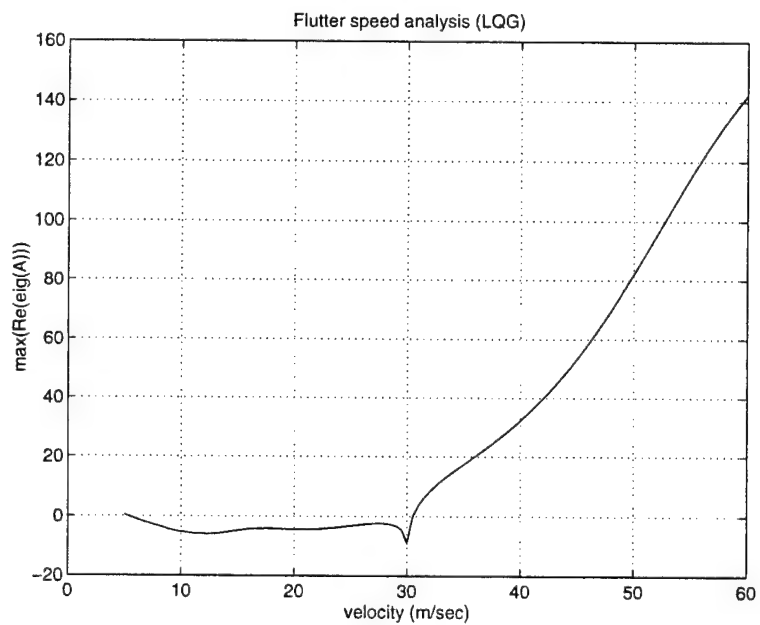
14

Figure 4: Flutter analysis (open loop)



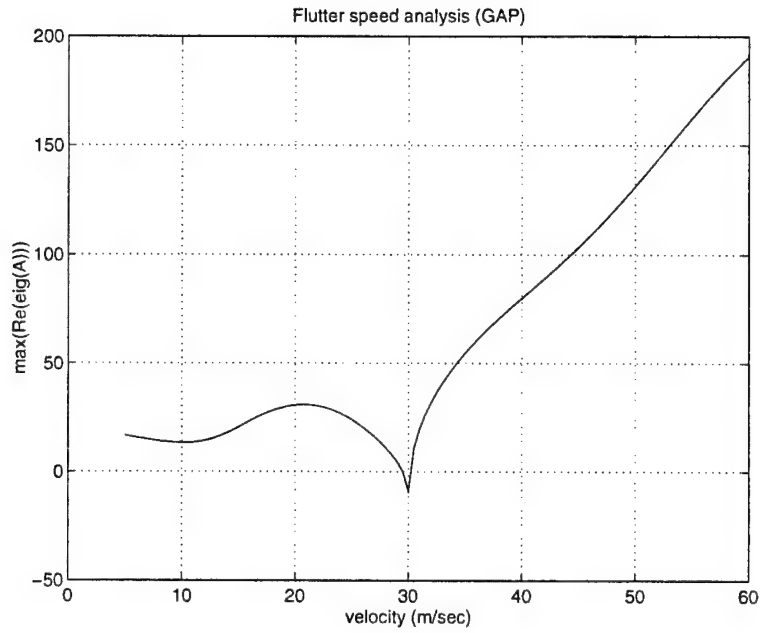Figure 5: Flutter analysis (LQG)
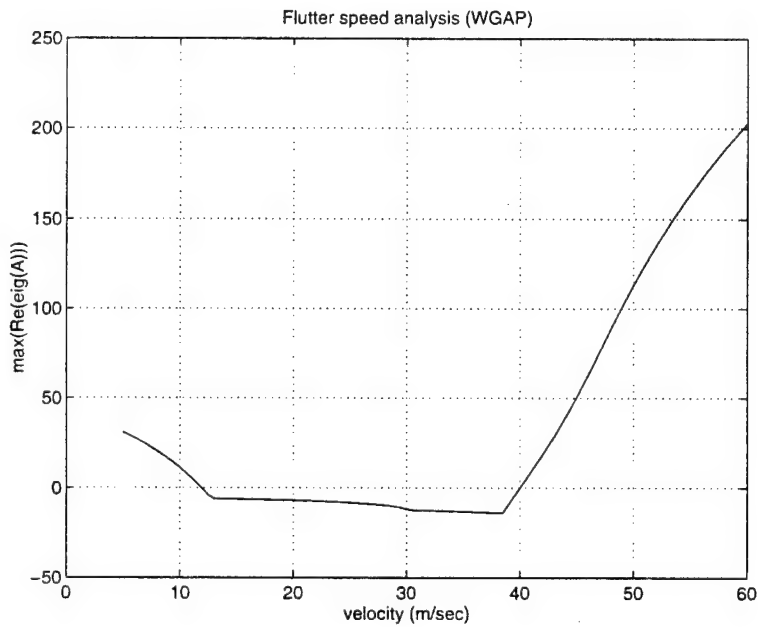
15

Figure 6: Flutter analysis (unweighted gap)



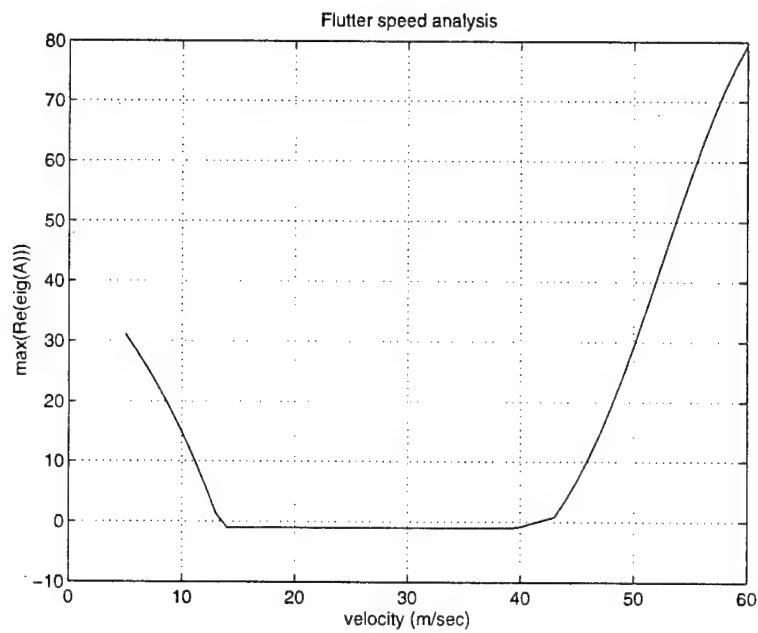Figure 7: Flutter analysis (weighted gap)

16

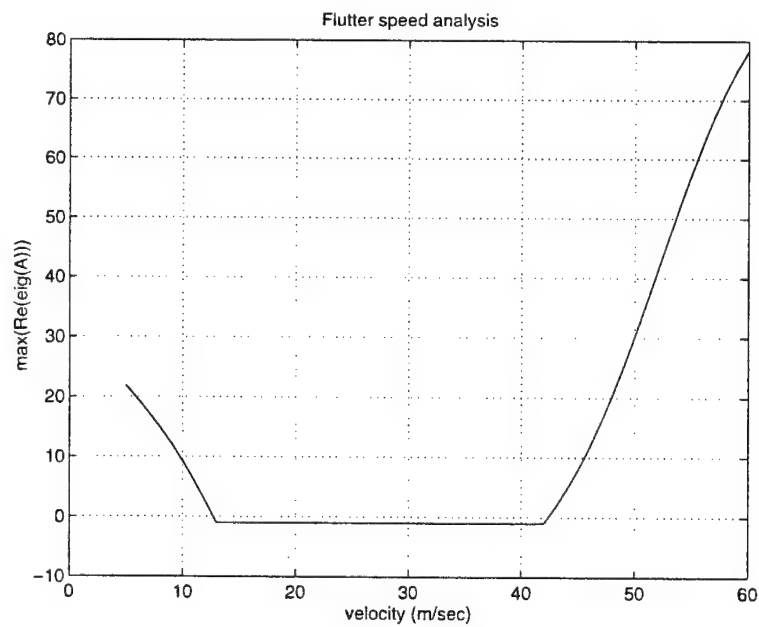Figure 8: Flutter analysis ($H^\infty$ mixed sensitivity minimization)



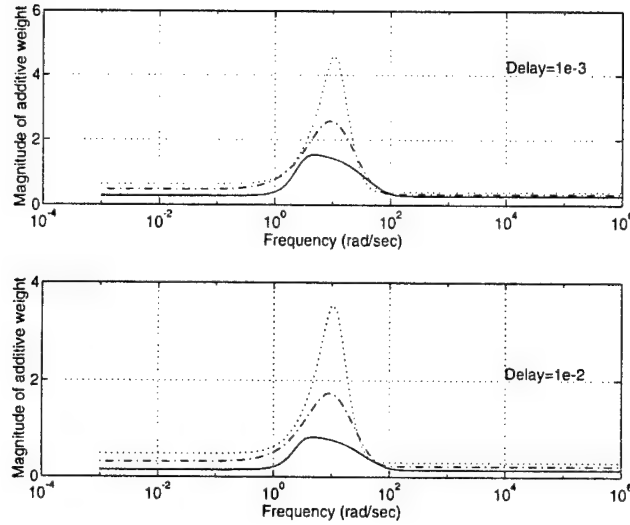Figure 9: Flutter analysis ($\mu$ synthesis)

17

Figure 10: Largest Allowable Additive Uncertainty

Of course, it is better to include the time delay in the plant model, if it is known. In this case a controller can be designed to achieve better performance and stability properties. Optimal robustness level $b_{opt}$, corresponding to this aeroelastic system (with transfer matrix $P(s) = e^{-Ts}1000P_o(s)$, where $P_o$ depends on the air speed $V$) for three different values of $V$, as a function of the time delay $T$ is shown in Figure 13. Since the unweighted gap does not give a reasonable robustness with respect to perturbations in $V$ we chose to design a weighted gap for the original plant scaled by 1000. Note that time delays in the order of 10msec leads to poor robustness. That is, $b_{opt}$ is considerably smaller when delay is around 10msec or higher. Whereas time delays up to 0.2msec do not affect the optimal robustness level significantly. The same figure also shows that as $V$ increases the optimal robustness level $b_{opt}$ decreases.

For the numerical values $V = 30m/sec$ $T = 1msec$, we find that optimal weighted gap robustness is $b_{max} = 0.32$. If we choose $b_{sub} = 0.30$ as the suboptimal robustness level, and apply the controller design algorithm we developed for MIMO unstable delay systems we obtain a controller $C(s)$ which gives $b_{P,C} = 0.30$. Note that in this numerical example the controller is a $2 \times 2$ transfer matrix of the form

$$C(s) = (D_n + B_n(sI - A_n)^{-1}C_n)\Big((D_d + B_d(sI - A_d)^{-1}C_d)$$
$$+ e^{-Ts}(D_{dd} + B_{dd}(sI - A_{dd})^{-1}C_{dd})\Big)^{-1}.$$

The MATLAB based program written for this problem generates the matrices $D_n$, $B_n$, $A_n$, $C_n$, $D_d$, $B_d$, $A_d$, $C_d$, $D_{dd}$, $B_{dd}$, $A_{dd}$, $C_{dd}$. In this example it turns out that
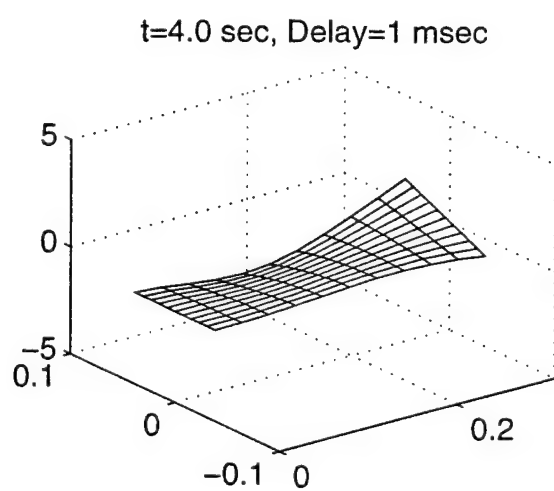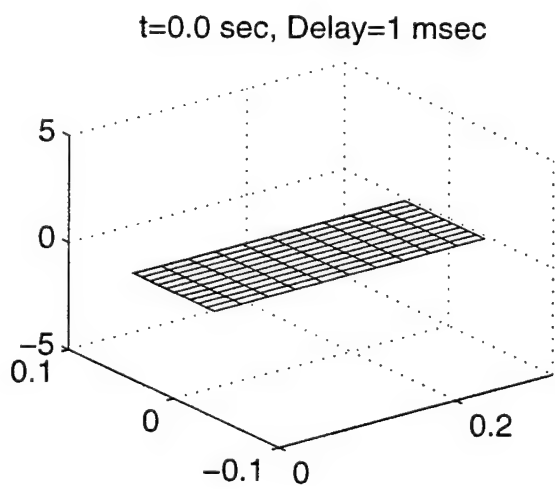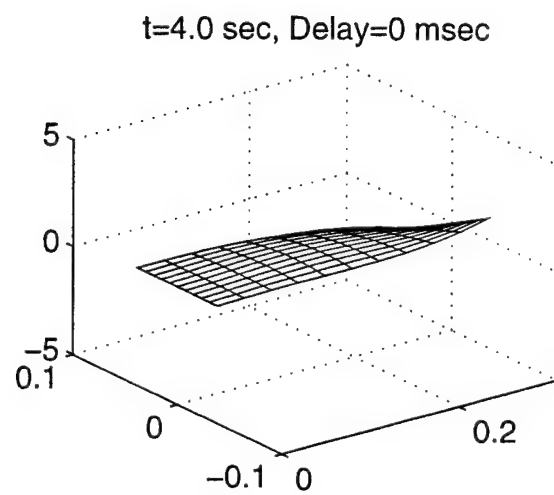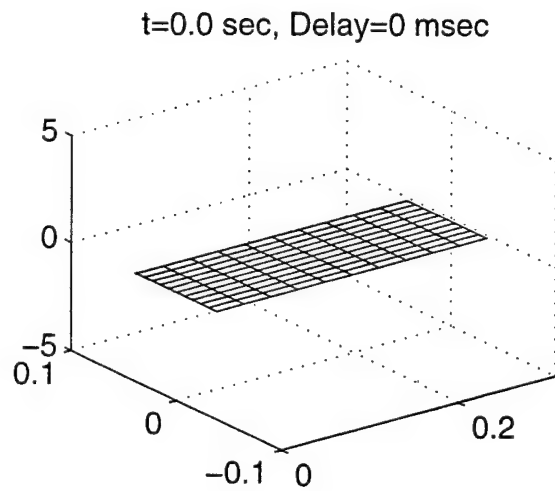
18

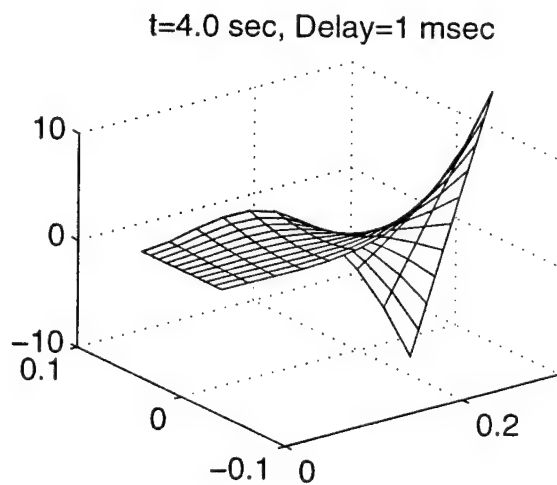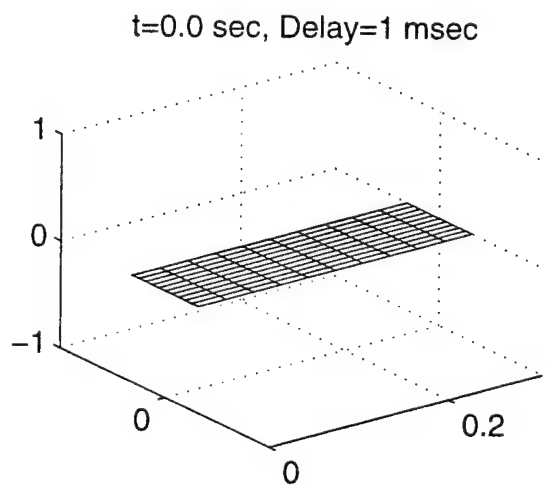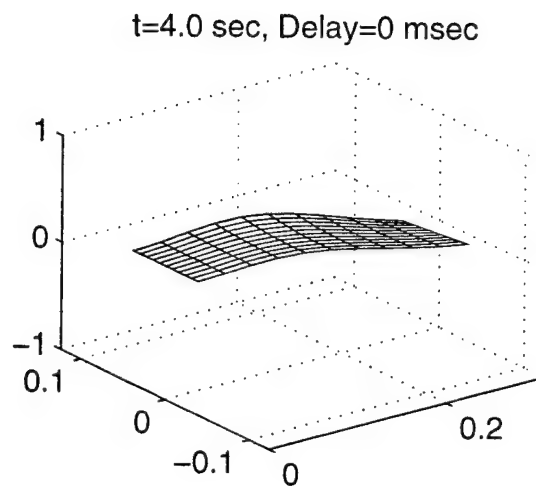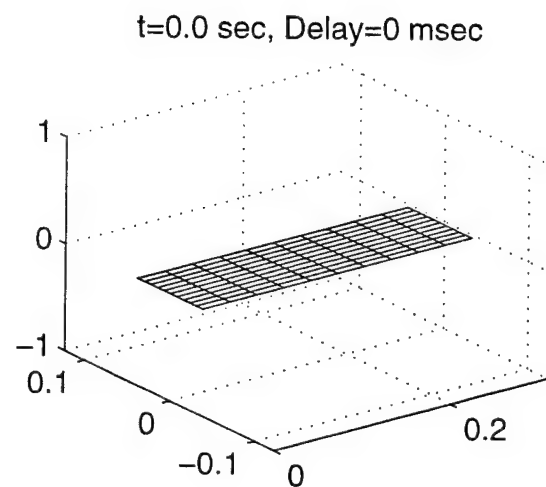Figure 11: Initial and final shape of the structure with $H^\infty$ control

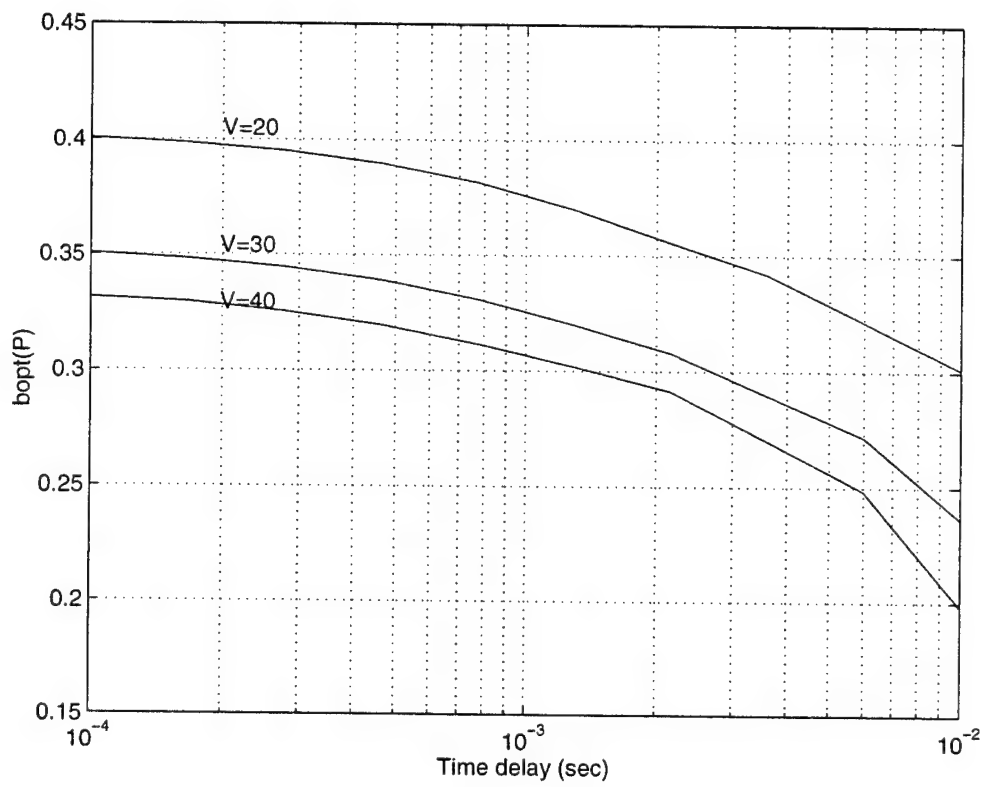Figure 12: Initial and final shape of the structure with LQG control

Figure 13: Optimal Robustness versus Time Delay

the matrices $A_n$, $A_d$ and $A_{dd}$ have dimensions $294 \times 294$, $236 \times 236$, and $114 \times 114$ respectively. Therefore, the finite dimensional parts of the controller is very high order. However, the magnitude and phase plots of the entries of the controller suggest that there should be a low order approximation. At this point any approximation algorithm can be used (see [11] and references therein) to implement a low order controller.

# 5   Remarks on the Computational Aspects

This section summarizes the results obtained, during the project, on computational aspects of certain robust control problems.

## 5.1   $H^\infty$ control of SISO infinite dimensional plants

Consider the closed loop system shown in Figure 15, where $P(s)$ and $C(s)$ are the transfer functions of the plant and the controller respectively. In this section we give a very simple expression for the $H^\infty$ optimal and suboptimal controllers. These controllers are computed by solving a *finitely many linear equations*, which can be written directly form the controller structure.

It is assumed that, the plant has finitely many unstable poles denoted by $\alpha_1, \ldots, \alpha_l$, and the transfer function can be factored as

$$P(s) = \frac{m_n(s)N_o(s)}{m_d(s)} \qquad \text{where} \qquad m_d(s) = \prod_{k=1}^{l} \frac{s - \alpha_k}{s + \alpha_k},$$

$m_n \in H^\infty$ is inner (i.e. all-pass function) possibly infinite dimensional, and $N_o \in H^\infty$ is outer (i.e. minimum phase) possibly infinite dimensional. It is also assumed that $\alpha_1, \ldots, \alpha_l \in \mathbb{C}_+$ are distinct. Let $S = (1 + PC)^{-1}$ and $T = 1 - S$, and $W_1$, $W_2$ be given weighting functions. The optimal mixed sensitivity problem is to find

$$\gamma_o := \inf_{C \text{ stabilizes } P} \left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty \tag{13}$$

and the optimal $H^\infty$ controller, denoted by $C_{opt}$, corresponding to the optimal perfor-
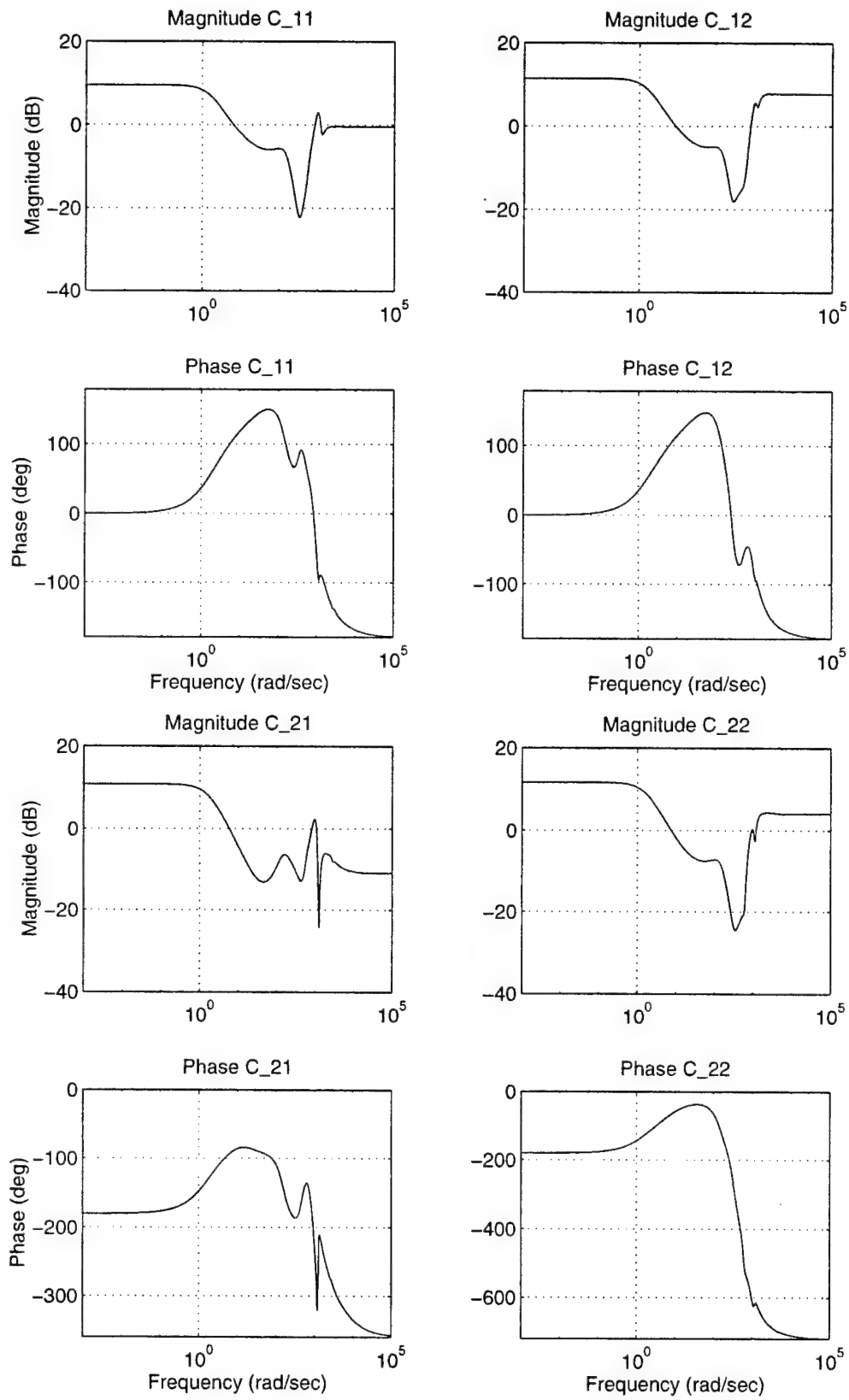
Figure 14: Magnitude and phase plots of the controller.
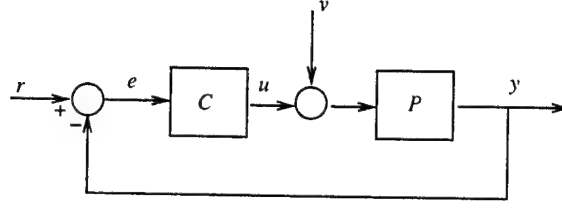
Figure 15: SISO Feedback System

mance level $\gamma_o$. That is, $C_{opt}$ stabilizes the plant $P$, and yields

$$
\left\| \begin{bmatrix} W_1(1 + PC_{opt})^{-1} \\ W_2PC_{opt}(1 + PC_{opt})^{-1} \end{bmatrix} \right\|_\infty = \gamma_o.
$$

The suboptimal mixed sensitivity problem is to parameterize the set

$$
\mathcal{C}_\rho = \left\{ C \quad : \quad C \text{ stabilizes } P, \quad \left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty \leq \rho \right\} \tag{14}
$$

for a given desired performance level $\rho > \gamma_o$.

In this section, it is assumed that $W_1(s)$ and $W_2(s)$ are rational functions. For properness of the optimal controller it will be assumed that $W_1(s)$ is non-constant and $W_1$, $(W_2N_o)$ and $(W_2N_o)^{-1} \in H^\infty$.

Let $\eta_1, \ldots, \eta_{n_1} \in \overline{\mathbb{C}}_+$, be the poles of $W_1(-s)$ (if $\eta_i$ has multiplicity $\ell_i$ then it is assumed to be repeated $\ell_i$ times in this list), and set

$$
E_\gamma(s) := \left( \frac{W_1(-s)W_1(s)}{\gamma^2} - 1 \right). \tag{15}
$$

The zeros of $E_\gamma(s)$ are denoted by $\beta_1, \ldots, \beta_{2n_1}$. Suppose they are distinct for a chosen $\gamma$. Then, $\beta_i$'s can be enumerated in such a way that $\beta_1, \ldots, \beta_{n_1}$ are in $\overline{\mathbb{C}}_+$, and $\beta_{n_1+i} = -\beta_i$. Now define

$$
F_\gamma(s) := G_\gamma(s) \prod_{k=1}^{n_1} \frac{\eta_k - s}{\eta_k + s} \tag{16}
$$

24

where $G_\gamma \in H^\infty$ is minimum phase and determined from the spectral factorization

$$G_\gamma(s)G_\gamma(-s) := \left(1 - \left(\frac{W_2(-s)W_2(s)}{\gamma^2} - 1\right)E_\gamma(s)\right)^{-1}. \tag{17}$$

Then (under certain genericity assumptions–see [28]) the optimal $H^\infty$ controller is given by

$$C_{opt}(s) = E_{\gamma_o}(s)m_d(s)\frac{N_o(s)^{-1}F_{\gamma_o}(s)L(s)}{1 + m_n(s)F_{\gamma_o}(s)L(s)} \tag{18}$$

where $L(s) = L_2(s)/L_1(s)$, and $L_1(s), L_2(s)$ are polynomials of degrees less than or equal to $(n_1 + l - 1)$ and they satisfy

$$0 = L_1(\beta_k) + m_n(\beta_k)F_{\gamma_o}(\beta_k)L_2(\beta_k) \qquad k = 1,\ldots,n_1 \tag{19}$$
$$0 = L_1(\alpha_k) + m_n(\alpha_k)F_{\gamma_o}(\alpha_k)L_2(\alpha_k) \qquad k = 1,\ldots,l \tag{20}$$
$$0 = L_2(-\beta_k) + m_n(\beta_k)F_{\gamma_o}(\beta_k)L_1(-\beta_k) \quad k = 1,\ldots,n_1 \tag{21}$$
$$0 = L_2(-\alpha_k) + m_n(\alpha_k)F_{\gamma_o}(\alpha_k)L_1(-\alpha_k) \quad k = 1,\ldots,l. \tag{22}$$

In general, if $\gamma_o$ is replaced by a variable, say $\gamma$, in equations (19–22), then a new set of linear homogeneous equations is obtained, in terms of $2(n_1 + l)$ unknown coefficients of $L_1$ and $L_2$, for each fixed $\gamma$. In [28] it was shown that, under certain genericity assumptions, $\gamma_o$ is the largest value of $\gamma$ for which there is a non-trivial solution to these $2(n_1 + l)$ linear homogeneous equations. That is, $\gamma_o$ can be found by plotting smallest singular values of the matrix representation of these equations, as $\gamma$ varies in an interval. The largest value of $\gamma$ for which the plot shows a zero is $\gamma_o$. It was also shown that, see [28], in the optimal case $L_2(s) = L_1(-s)$ and hence the number of unknown coefficients in these coefficients reduce to $n_1 + l$; and these can be determined from (19,20).

Similarly, all suboptimal $H^\infty$ controllers are in the form

$$C_{subopt}(s) = E_\rho(s)m_d(s)\frac{N_o(s)^{-1}F_\rho(s)L_U(s)}{1 + m_n(s)F_\rho(s)L_U(s)} \tag{23}$$

where

$$L_U(s) = \frac{L_2(s) + L_1(-s)U(s)}{L_1(s) + L_2(-s)U(s)} \quad , \quad U \in \mathcal{B},$$

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and $L_1(s), L_2(s)$ are polynomials of degree $\leq n_1 + l$ satisfying (19-22) with $\gamma_o$ replaced by $\rho$, and the following two conditions:

$$0 = L_2(-a) + (E_\rho(a) + 1)F_\rho(a)m_n(a)L_1(-a) \tag{24}$$

$$1 = L_1(-a), \tag{25}$$

for some arbitrary $a \in \mathbf{R}$, $a > 0$, $a \neq \alpha_i$'s and $a \neq \beta_j$'s for all $i = 1, \ldots, l$ and $j = 1, \ldots n_1$. In (24,25) $a$ is a free parameter. For different values of $a$ one can obtain different parameterizations of the suboptimal controllers.

## 5.2   Finite Dimensional $H^\infty$ Controllers via Approximations

The structure of $H^\infty$ controllers (optimal and suboptimal) is given in the previous section, and it is shown that for distributed systems, the $H^\infty$ optimal controller is not rational. Therefore, the question of obtaining a stabilizing finite dimensional $H^\infty$ controller, whose performance is "close" to optimum, arises naturally. If a controller satisfies this requirement, then it is said to be "approximately optimal." One can approximate the infinite dimensional parts of the optimal controller to obtain such a controller, and this method has been studied before [17] for stable plants. For unstable plants it was shown that [14] if coprime factors of the optimal controller are approximated separately, then under mild conditions, the resulting finite dimensional controller is approximately optimal. However, computation of the coprime factors of the optimum controller requires construction of a complicated system of linear equations, (compare the formulae of the previous section with the ones in [16, 14]).

One possible approach to obtain a finite dimensional controller $C_f$ from the controller $C$ given in (18) is to approximate the infinite dimensional parts of the controller, $N_o^{-1}$ and $m_n$, by rational functions, $N_{of}^{-1}$ and $m_{nf}$, and obtain

$$C_f = E_{\gamma_o}(s)m_d(s)\frac{N_{of}^{-1}(s)F_{\gamma_o}(s)L(s)}{1 + m_{nf}(s)F_{\gamma_o}(s)L(s)}. \tag{26}$$

Define

$$S_f = (1 + PC_f)^{-1}, \quad T_f = 1 - S_f, \ S_{opt} = (1 + PC_{opt})^{-1}, \quad T_{opt} = 1 - S_{opt},$$

$$\Delta_{m_n} = \frac{F_{\gamma_o}L}{1 + m_n F_{\gamma_o}L}(m_{nf} - m_n), \quad \Delta_{N_o} = 1 - N_{of}^{-1}N_o.$$

The following lemma gives a set of sufficient conditions for the closed loop system $(P, C_f)$ to be stable.

**Lemma 1:** The closed loop system $(P, C_f)$ is stable if

(i) $m_{nf}$ interpolates $m_n$ at $\alpha_i$'s, $\beta_j$'s in $\mathbb{C}_+$, and $j\omega$ axis zeros of $L_1 + m_n F_\gamma L_2$,

(ii) $\quad \sup\limits_{\omega \in \mathbb{R}} \left| \Delta_{m_n} \right| < 1,$

(iii) $\quad \sup\limits_{\omega \in \mathbb{R}} \left( |\gamma_o W_2^{-1}(j\omega) \Delta_{N_o}(j\omega)| + |\gamma_o W_2^{-1}(j\omega) N_{of}^{-1}(j\omega) N_o(j\omega) \Delta_{m_n}(j\omega)| \right) < 1,$

(iv) $1 - N_{of}^{-1} N_o \in H^\infty.$

Now, let us consider the performance of the closed loop system $(P, C_f)$.

**Lemma 2:** The following conditions guarantee that the performance $\gamma(P, C_f)$ of the closed loop system $(P, C_f)$ will be between $\gamma_o$ and $\gamma_o + \epsilon$:

(i) $\quad \sup\limits_{\omega \in \mathbb{R}} |W_1^{-1}(j\omega) \Delta_{N_o}(j\omega)| < \dfrac{\epsilon}{4\gamma_o^2}(1 - \epsilon),$

(ii) $\quad \sup\limits_{\omega \in \mathbb{R}} |W_1^{-1}(j\omega) \Delta_{m_n}(j\omega)| < \dfrac{\epsilon}{4\gamma_o^2}(1 - \epsilon),$

(iii) $\quad \sup\limits_{\omega \in \mathbb{R}} |W_2^{\prime -1}(j\omega) \Delta_{N_o}(j\omega)| < \dfrac{\epsilon}{4\gamma_o^2}(1 - \epsilon), \quad$ and

(iv) $\quad \sup\limits_{\omega \in \mathbb{R}} |W_2^{-1}(j\omega) \Delta_{m_n}(j\omega)| < \dfrac{\epsilon}{4\gamma_o^2}(1 - \epsilon),$

where $\epsilon$ is a positive real number less than 1.

Note that, since $\frac{F_{\gamma_o} L}{1 + m_n F_{\gamma_o} L}$ strictly proper, approximation conditions on $m_{nf}$ can be interpreted as a uniform approximation in the low frequency range subject to certain interpolation conditions in the closed right half plane. Interpolating both the values of $m_n$ and its derivatives at $\xi_i$'s gives better $m_{nf}$'s functions which is likely to decrease the value of the expression

$$\left| \frac{(m_n(j\omega) - m_{nf}(j\omega)) F_\gamma(j\omega) L(j\omega)}{1 + m_n(j\omega) F_\gamma(j\omega) L(j\omega)} \right|,$$

27

around $\xi_i$'s. Note that the supremum of this expression over $j\omega$ axis appears as one of the conditions of Lemma 1.

We refer to [23] and [29] for a numerical example illustrating the approximation procedure proposed in this section.

## 5.3  On the NP-hardness of certain control problems

In this section we simply state the main results of the papers [27] and [25], on the computational complexity of certain robust control problems. The reader is refered to the full versions of these papers for complete details.

In [27], it is shown that for a given complex matrix $M$, and a purely complex uncertainty structure $\boldsymbol{\Delta}$, the problem of checking whether the inequality

$$(\min\{\sigma_{max}(\Delta) \ : \ \Delta \in \boldsymbol{\Delta}, \ \ \det(I - M\Delta) = 0\})^{-1} =: \mu_{\boldsymbol{\Delta}}(M) < 1$$

holds, is $\mathcal{NP}$-hard. It is also shown that, the problem of checking whether the frequency domain $\mu$, $\|M(s)\|_\mu$, of an LTI system, $M(s)$, is less than 1, and the problem of checking whether the best achievable $\mu$,

$$\inf_{Q \in H^\infty} \|\mathcal{F}(T, Q)\|_\mu,$$

of a linear fractional transformation (LFT), $\mathcal{F}(T, Q)$, is less than one, are both $\mathcal{NP}$-hard problems. In other words, purely complex $\mu$ computation, analysis/synthesis are $\mathcal{NP}$-hard.

It was known that the computation of $\mu$ is $\mathcal{NP}$-hard for purely real [20] and mixed real/complex [3] uncertainty structures, but these results do not give much information about the computational complexity of the purely complex $\mu$ problem, nor they imply much about complexity of the $\mu$ analysis/synthesis problems. Although general $H^\infty$ norm computation, analysis/synthesis have a well established theory for LTI systems, there is no known non-conservative polynomial time procedure for purely complex $\mu$ computation, analysis/synthesis problems. In this part of the project, [27], we gave proofs of the $\mathcal{NP}$-hardness of the above mentioned problems. These results imply that it is rather unlikely to find non-conservative polynomial time procedures for the purely complex $\mu$ computation, analysis/synthesis problem, contrary to the standard $H^\infty$ problems.

As independent results, it is also shown that, [27], the problem of checking the stability and the problem of computing the $H^\infty$ norm, are both $\mathcal{NP}$-hard problems for multidimensional systems. These results imply that it is rather unlikely to find a simple analogue of the Schur-Cohn test for checking the stability and an efficient generalization of bisection method of [2] for computing the $H^\infty$ norm, in the context of multidimensional systems.

A matrix valued function, $F(X, Y)$, is called bilinear if it is linear with respect to each of its arguments. An inequality of the form $F(X, Y) > 0$ is called a bilinear matrix inequality (BMI). Recently it has been shown that many robust control problems can be transformed to finding a solution to bilinear matrix inequalities (BMIs). For example the static output feedback problem and fixed order robust controller design problem can be put in this framwork. In [25] we have shown that the problem of checking the solvability of a given BMI is NP-hard. Although BMI approaches to robust control seems to be a potentially powerful tool, it is rather unlikely to find a polynomial time algorithm for solving general BMI problems. In [25] it is also shown that simultaneous stabilization with static output feedback is an NP-hard problem, namely given $n$ plants, the problem of checking the existence of a static gain matrix $K$ which stabilizes all of the $n$ plants, is NP-hard.

## 5.4   On the order of simultaneously stabilizing compensators

In this project the problem of simultaneous stabilization using a dynamic compensator has also been studied. The results will appear in [24], where it is shown that there is no upper bound for the minimal order of a simultaneously strongly stabilizing compensator, in terms of the given plant orders. A similar problem was also considered in [21], where it was shown that such a bound does not exist for the strong stabilization problem of a single plant. But the examples given in [21] were *forcing* an approximate unstable pole-zero cancellation, or *forcing* the distance between two distinct unstable zeros to go zero. In [24] it is shown that: (i) if approximate unstable pole-zero cancellation does not occur, and the distances between distinct unstable zeros are bounded below by a positive constant, then it is possible to find an upper bound for the minimal order of a strongly stabilizing compensator; (ii) and for the simultaneous strong stabilization problem (even for the two plant case), such a bound cannot be found.

# 6 Conclusions

In this project robust control techniques are investigated for unstable aeroelastic systems. A new algorithm is developed (and its MATLAB based program is written) for MIMO delay systems to optimize robustness measured in the gap. The algorithm can easily be modified to obtain controllers from weighted gap optimization method. Numerical simulations have shown that when time delays are neglected in the design of LQG, gap, loop shaping via weighted gap, $H^\infty$, and $\mu$ controllers, the closed loop stability may be lost or the performance may be poor as far as flutter suppression problem for a thin airfoil is concerned. Our algorithm has been tested on the same example, and an approximable infinite dimensional controller is obtained.

In the SISO case numerical computation of $H^\infty$ controllers is simplified considerably. A simple MATLAB based program is written to compute the optimal performance, optimal controller, and all suboptimal controllers. Also, a numerical approximation procedure is developed to derive finite dimensional suboptimal controllers for general infinite dimensional plants.

Computational complexity of certain control algorithms are studied. In particular it is shown that purely complex $\mu$ computation, analysis and synthesis problems are NP-hard; and so are the problems of simultaneous stabilization with static output feedback and solution to bilinear matrix inequalities. It is important to note that the above problems have been considered by many researchers in the past, and no polynomial time algorithmic procedure has been developed for exact computation. The results of this research indicate that it is practically impossible to find a polynomial time exact computation procedure for the above mentioned problems. So, the scientific community should be satisfied with approximate solutions, which are computationally more feasible.

As far as the aeroelasticity application (flutter suppression), simulations show that $H^\infty$ based controllers give good robustness and performance compared to more conventional LQG method. Of course in both cases weight selection is a major issue. Even though we now have tools to compute $H^\infty$ and LQG controllers, in many cases for a given set of weights and the plant the "central controller" is unstable (see [15] for a aircraft pitch angle command tracking, and gust alleviation examples). On the other hand for safety reasons, in the actual implementation, unstable controllers are undesirable. Therefore, it would be a major contribution if one develops methods to determine stable suboptimal controllers. The PI will be focusing on this problem in the future.

# 7 List of Publications Based on the Project

1. O. Toker and H. Özbay, "Gap metric problem for MIMO delay systems: Parameterization of all suboptimal controllers," *Automatica*, to appear.

2. O. Toker and H. Özbay, "$H^\infty$ Optimal and suboptimal controllers for infinite dimensional SISO plants," *IEEE Transactions on Automatic Control*, to appear.

3. C. Gu, O. Toker and H. Özbay, "On the two block $H^\infty$ problem for a class of unstable distributed systems," *Linear Algebra and its Applications*, to appear.

4. O. Toker, "On the order of simultaneously stabilizing compensators," *IEEE Transactions on Automatic Control*, to appear.

5. O. Toker and H. Özbay, "On the NP-hardness of solving bilinear matrix inequalities and simultaneous stabilization with static output feedback," *Proceedings of the American Control Conference*, Seattle WA, 1995, to appear.

6. O. Toker and H. Özbay, "Robustness analysis of controllers designed for active flutter suppression," *Proceedings of the American Control Conference*, Seattle WA, 1995, to appear.

7. O. Toker and H. Özbay, "On the NP-hardness of the purely complex mu computation, analysis/synthesis, and some related problems in multidimensional systems," *Proceedings of the American Control Conference*, Seattle WA, 1995, to appear.

8. O. Toker and H. Özbay "On the structure of $H^\infty$ controllers," *Proceedings of 33rd IEEE Conference on Decision and Control*, Lake Buena Vista, December 1994, pp. 3949–3954.

9. O. Toker and H. Özbay, "Numerical issues in the computation of $H^\infty$ controllers for distributed plants," *Proceedings of 31st Allerton Conference on Communication, Control and Computing*, University of Illinois Urbana-Champaign, September-October 1993, pp. 535–544.

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